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On the LIDS of corona product of graphs

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Abstract. Let $G = (V, E)$ be a simple, undirected, and nontrivial graph. An independent set is a set of vertices in a graph in which no two of vertices are adjacent. A dominating set of a graph G is a set D of vertices of G such that every vertex not in S is adjacent to a vertex in D. An independent dominating set in a graph is a set that is both dominating and independent. Equivalently, an independent dominating set is a maximal independent set. Locating independent dominating set of graph G is independent dominating set with the additional characteristics that for $u, v \in (V(G) - D)$ satisfies $N(u) \cap D \neq N(v) \cap D$. $\gamma_{Li}(G)$ is the minimum cardinality of locating dominating set we call Locating domination number. In this paper, we analyze the locating independent domination number of corona product of path, cycle, gear, wheel, and ladder graph. We also analyze whether locating independent domination number of corona product depends on its constituent graphs.

INTRODUCTION

Definition about graph defined by [6, 2]. Let $G = (V, E)$ be a simple, undirected, and nontrivial graph. For $v \epsilon V(G)$, the neighborhood $N_G(v)$ (or simply $N(v)$) of v is the set of all vertices adjacent to v in G.

A set D of vertices of a graph $G = (V, E)$ is dominating if every vertex in $V(G) - D$ is adjacent to some vertex in D. A non-empty set $D \subseteq V$ of a graph G is a dominating set, if every vertex in $V - D$ is adjacent to at least one vertex in D. $\gamma(G)$ is the minimum cardinality of a dominating set in G we call domination number. Waspodo *et. al.* [4] determined the lower bound of distance domination number of $(G \trianglerighteq H)$ and Dafik *et. al.* [10] studied about locating domination number of $S_n \trianglerighteq H$.

Let D be a dominating set of G, if $N[v] \cap D \neq \emptyset$ for all vertex $v \in G$, or equivalently, $N[D] = V(G)$. A dominating set $D \subseteq V$ is called a locating dominating set (LDS), if for any two vertices $v, w \epsilon V - D$, $N(v) \cap D \neq N(w) \cap D$. $\gamma_L(G)$ and $\Gamma_L(G)$ is the minimum and maximum cardinality of a minimal LDS of G we call the locating domination number and the upper locating domination number. LDS of minimum cardinality is called a $\gamma_L(G)$-set, and we define a $\Gamma_L(G)$-set likewise. Slater [3, 5, 7, 8] firstly studied the concept of LDS. The independent domination number $i(G)$ is the minimum number of vertices in an independent set $D \subseteq V(G)$ such that D also dominates $V(G)$. The lower independence number or the independent domination number is the minimum cardinality of a maximal independent set. Locating independent dominating set (LIDS) is special case with the additional characteristics that D is an independent set and every vertex not in D is adjacent to a vertex in D. The minimum cardinality of LIDS is locating independent domination number, denoted by $\gamma_{LI}(G)$. LIDS of order $\gamma_{LI}(G)$ is called an $\gamma_{Li}(G)$-set. Wardani [11] determined the lower bound of locating independent domination number of Amal(G, v, m). For definition and notation of LDS and LIDS defined by [1].

In this research we determine the lower bound of γ_{Li} of corona graph $G \odot H$. Corona product of graphs G_1 and G_2, is the graph which is the disjoint union of one copy of G_1 and $|V_1|$ copies of G_2 ($|V_1|$ is the number of vertices of G_1) in which each vertex of the copy of G_1 is connected to all vertices of a separate copy of G_2. The definition of corona graph is taken from [12]. Let two graphs G and H is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the i-th vertex of G to every vertex in the i-th copy of H.

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MAIN RESULT

In this section, we determine the lower bound of locating independent domination number of corona product of graph $G \odot H$.

Lemma 0.1. *G, H be a simple, connected, and undirected graphs. If $G \odot H$, then locating independent dominating set of $G \odot H$ is located in H.*

Proof. The corona graph $G \odot H$ is a connected graph with vertex set $V(G \odot H) = V(G) \cup \{w_{i,j}; 1 \leq i \leq |V(H)|; 1 \leq j \leq |V(G)|\}$ and edge set $E(G \odot H) = E(G) \cup E(H) \cup V(G_i)V(H_i)$. The order and size of order $(G \odot H)$ are $|V(G \odot H)| = p(G) + p(H)p(G)$ and size $|E(G \odot H)| = q(H) + p(H)p(G) + q(H)$.

Let D be an LIDS. Suppose $D \subseteq V(G)$, $\forall d \in D$ will have neighbor then $\forall h \in H_i$, for H_i which incident with d. Furthermore, $\exists d_1, d_2 \in D$ which are adjacent. It is contradiction, $D \subseteq V(H)$. □

Lemma 0.2. *For any graph G of order n and H of order m, so*

$$\gamma_{Li} \geq \begin{cases} p(G)(\gamma_{L_i}(H)) & ;\text{for } diam(H) > 2 \\ \sim & ;\text{for } diam(H) \leq 2; \gamma_{Li}(H) = \sim \end{cases}$$

Proof. Case 1. The graph $G \odot H$ have n-subgraph H_i for $1 \leq i \leq n$. Base on 0.1 the dominator located in $V(H_i)$, thus $|D(H_1)| = |D(H_2)| = |D(H_3)| = \ldots = |D(H_n)|$. Such that,
$|D(G \odot H)| \geq |D(H_1)| + |D(H_2)| + |D(H_3)| + \ldots + |D(H_n)|$
$|D(G \odot H)| \geq n|D(H_i)|$
$|D(G \odot H)| \geq p(G)\gamma_{Li}(H)$

Case 2. Base on 0.1 the dominator located in $V(H_i)$ and $\gamma_{Li}(H) = \sim$, thus $\gamma_{L_i}(G \odot H) = \sim$. □

Figure 1 is an example of corona graph and figure 2 is an example of LIDS of corona graph.

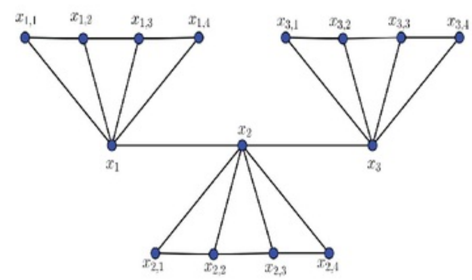

Figure 1. Corona of P_3 and P_4

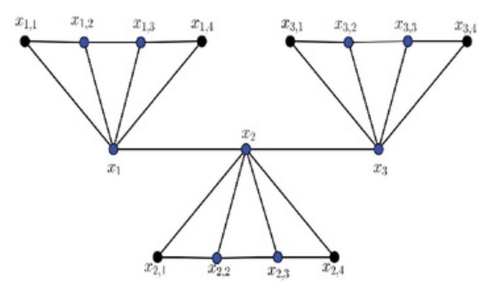

Figure 2. Locating Independent Dominating Set of $P_3 \odot P_4$

Theorem 0.3. *For $n \geq 3$ and $m \geq 4$, so $\gamma_{Li(W_n \odot P_m)} = (n+1)(\lceil \frac{2n}{5} \rceil)$.*

Proof. Corona graph $W_n \odot P_m$ is a connected graph with vertex set $V(W_n \odot P_m) = \{A\} \cup \{x_i; i = 1..n\} \cup \{x_{i,j}; i = 1..n+1; j = 1..m\}$ and edge set $E(W_n \odot P_m) = \{Ax_i; i = 1..n\} \cup \{x_i x_{i+1}; i = 1..n-1\} \cup \{x_n x_1\} \cup \{x_{i,j} x_{i,j+1}; i = 1..n+1; j = 1..m-1\} \cup \{x_i x_{i,j}; i = 1..n; j = 1..m\} \cup \{Ax_{i,j}; i = n+1; j = 1..m\}$. The order and size of $W_n \odot P_n$ are $|V(W_n \odot P_m)| = nm + m + n + 1$, $|E(W_n \odot P_m)| = 2nm + n + 2m - 1$, and $diam(P_m) = m - 1$. $\gamma_{Li}(P_m) = \lceil \frac{2m}{5} \rceil$

Base on Lemma 0.2 $\gamma_{Li}(W_n \odot P_m) \geq (n+1)(\lceil \frac{2n}{5} \rceil)$. We see that $\gamma_{Li}(W_n \odot P_m) \leq (n+1)(\lceil \frac{2n}{5} \rceil)$. Choose $D = \{x_{i,j}; i = 1..n+1; j$ is odd $\}$ as the dominator set of $W_n \odot P_m$), for $n \geq 3$ and $m \geq 4$, thus $|D| = (n+1)(\lceil \frac{2n}{5} \rceil)$. Choose the vertices of graph but non dominator set of graph$= \cup \{x_i; i = 1..n\} \cup \{x_{i,j}; i = 1..n+1; j$ is even $\}$. Furthermore, we will obtain the intersection among the neighborhood of vertex with vertex in vertex set of graph but non dominator set of graph and dominator set D in the following.

$$N(A) \cap D = \{x_i; 1 = n+1; j \equiv 1 \mod 2\}$$
$$N(x_i) \cap D = \{x_{i,j}; \leq i \leq n; j \equiv 1 \mod 2\}$$
$$N(x_{i,j}; j \equiv 0 mod 2) \cap D = \{x_{i,j}; \leq i \leq n+1; j \equiv 1 \mod 2\}$$

It can be show that the intersection are all different, and it is not empty set. It can be conclude that for $\gamma_{Li}(W_n \odot P_m) \leq (n+1)(\lceil \frac{2n}{5} \rceil)$, it will observe the condition of LIDS. Thus $\gamma_{Li}(W_n \odot P_m) \leq (n+1)(\lceil \frac{2n}{5} \rceil)$. Therefore, $\gamma_{Li}(W_n \odot P_m) = (n+1)(\lceil \frac{2n}{5} \rceil)$. □

Figure 3 is corona graph of wheel graph and path graph. Figure 4 is LIDS of corona of wheel graph and path graph.

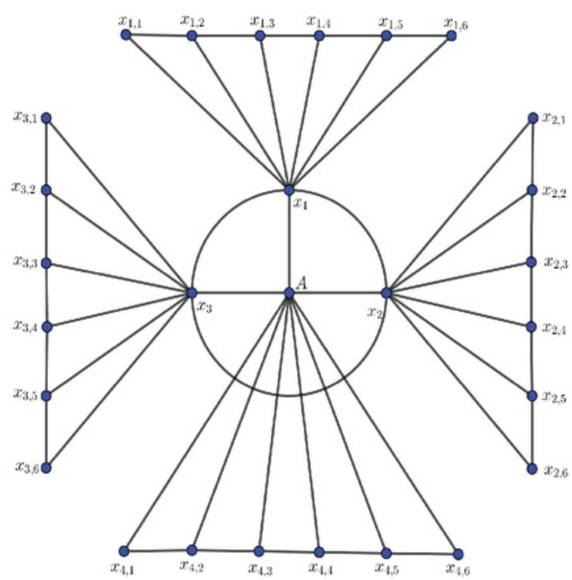

Figure 3. Corona of W_3 and P_6

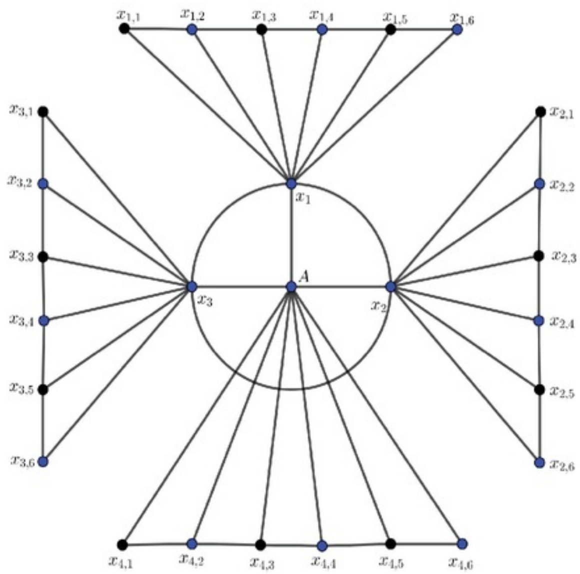

Figure 4. Locating Independent Dominating Set of $W_3 \odot P_6$

Theorem 0.4. *For* $m \geq 4$, *so* $\gamma_{Li(K_n \odot L_m)} = nm$.

Proof. Corona graph $K_n \odot L_m$ is a connected graph with vertex set $V(K_n \odot L_m) = \{x_i; i = 1..n\} \cup \{x_{i,j}; i = 1..n; j = 1..m\} \cup \{y_{i,j}; i = 1..n; j = 1..m\}$ and edge set $E(K_n \odot L_m) = \{x_i x_i; i = 1..n\} \cup \{x_i x_{i+1}; i = 1..n-1\} \cup \{x_n x_1\} \cup \{x_{i,j} x_{i,j+1}; i = 1..n; j = 1..m-1\} \cup \{y_{i,j} y_{i,j+1}; i = 1..n; j = 1..m-1\} \cup \{x_{i,j} x_i; i = 1..n; j = 1..m\} \cup \{y_{i,j} x_i; i = 1..n; j = 1..m\} \cup \{x_{i,j} y_{i,j}; i = 1..n; j = 1..m\}$. The order and size of $K_n \odot L_m$ are $|V(K_n \odot L_m)| = n + 2nm$, $|E(K_n \odot L_m)| = \frac{n(n-1)}{2} + 5nm - 2n$, and $diam(L_m) = m.\gamma_{Li}(L_m) = m$.

Base on Lemma (1) $\gamma_{Li}(K_n \odot L_m) \geq nm$. We see that $\gamma_{Li}(K_n \odot L_m) \leq nm$. Choose $D = \{x_{i,j}; i = 1..n; j$ is odd $\} \cup \{y_{i,j}; i = 1..n; j$ is even $2\}$ as the dominator set of $K_n \odot L_m$, for $m \geq 4$, thus $|D| = nm$. Choose the vertices of graph but non dominator set of graph$= \{x_i; i = 1..n\} \cup \{x_{i,j}; i = 1..n; j$ is even $\} \cup \{y_{i,j}; i = 1..n; j$ is odd $\}$ as the non-dominator set of $K_n \odot L_m$) for $m \geq 4$. Furthermore, we will obtain the intersection among the neighborhood of vertex with vertex in vertex set of graph but non dominator set of graph and dominator set D in the following.

$N(x_i) \cap D = \{x_{i,j}; 1 \leq i \leq n; j \equiv 1 \mod 2\} \cup \{y_{i,j}; 1 \leq i \leq n; j \equiv 0 \mod 2\}$
$N(x_{i,j}) \cap D = \{x_{i,j-1}, x_{i,j+1}, y_{i,j}\}; 2 \leq j \leq n-2; i =$ even
$N(x_{i,n}) \cap D = \{x_{i,n-1}, y_n\}; n =$ even
$N(y_i i, 1) \cap D = \{x_{i,1}, y_{i,2}\}$
$N(y_i) \cap D = \{x_{i,j}, y_{i,j-1}, y_{i,j+1}\}; 3 \leq i \leq n; i =$ odd
$N(y_{i,n}) \cap D = \{x_{i,n}, y_{i,n-1}\}; n =$ odd

It can be show that the intersection are all different, and it is not empty set. It can be conclude that for $\gamma_{Li}(K_n \odot L_m) \leq nm$, it will observe the condition of LIDS. Thus $\gamma_{Li}(K_n \odot L_m) \leq nm$. Therefore, $\gamma_{Li}(K_n \odot L_m) = nm$. □

Figure 5 is corona graph of complete graph and ladder graph. Figure 6 is LIDS of corona of complete graph and ladder graph.

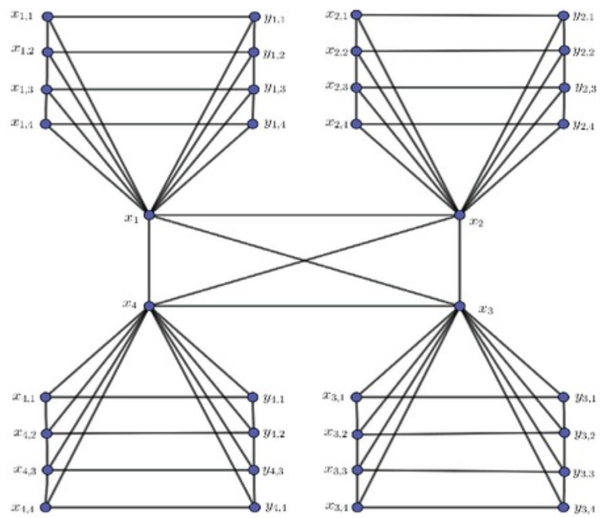

Figure 5. Corona of K_4 and L_4

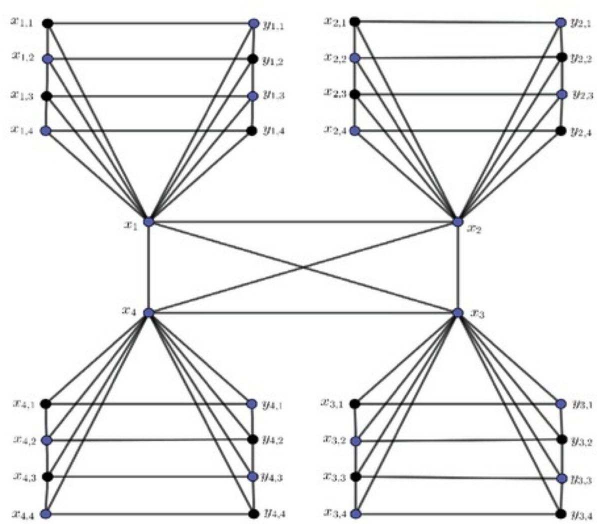

Figure 6. Locating Independent Dominating Set of $K_4 \odot L_4$

Theorem 0.5. *For* $n \geq 3$*, so* $\gamma_{Li(K_n)} = \sim$.

Proof. Complete graph K_n is a connected graph with order and size of K_n are $|V(K_n)| = n$, $|E(K_n)| = \frac{n(n+1)}{2}$, and $diam(K_n) = 1$. Base on Lemma 0.2 $\gamma_{Li}(L_n \odot K_m) = \sim$. □

Figure 7 is complete graph and its LIDS.

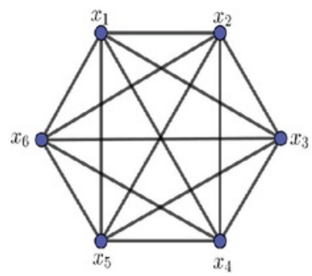

Figure 7. Complete Graph K_6

Theorem 0.6. *For* $n \geq 3$ *and* $m \geq 3$*, so* $\gamma_{Li(L_n \odot K_m)} = \sim$.

Proof. Corona graph $L_n \odot K_m$ is a connected graph with vertex set $V(L_n \odot K_m) = \{y_i; i = 1..n\} \cup \{x_i; i = 1..n\} \cup \{x_{i,j}; i = 1..n; j = 1..m\}$ *cup* $\{y_{i,j}; i = 1..n; j = 1..m\}$ and edge set $E(L_n \odot K_m) = \{x_i x_{i+1}; i = 1..n-1\} \cup \{y_i y_{i+1}; i = 1..n-1\} \cup \{x_i y_i; i = 1..n\} \cup \{x_{i,j} x_i; i = 1..n; j = 1..m\} \cup \{y_{i,j} y_i; i = 1..n; j = 1..m\} \cup \{x_{i,j} x_{i,j}; i = 1..n; j = 1..m\}$ *cup* $\{y_{i,j} y_{i,j}; i = 1..n; j = 1..m\}$. The order and size of $L_n \odot K_m$ are $|V(L_n \odot K_m)| = 2(n + nm)$, $|E(L_n \odot K_m)| = 3n + 2nm - 2 + \frac{m(m+1)}{2}$, and $diam(K_m) = 1$. Base on Lemma 0.2 $\gamma_{Li}(L_n \odot K_m) = \sim$. □

Figure 8 is corona graph of ladder graph with complete graph and its LIDS.

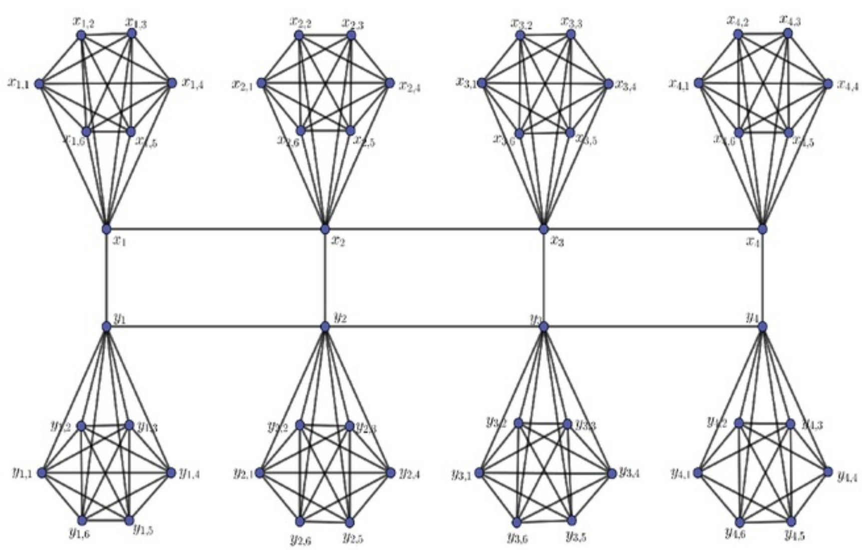

Figure 8. Corona of L_4 and K_6

CONCLUDING REMARKS

In this study, we given some result of locating independent domination number of corona graph $G \odot H$. Thus, it still gives the following open problem.

Open Problem 0.7. *For diam(H) ≤ 2, and H have γ_{Li}, determine some classes of graphs which doesn't have $\gamma_{Li}(G \odot H)$ in $G \odot H$?*

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