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The rainbow vertex antimagic coloring of tree graphs

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Abstract. Let $G(V(G), E(G))$ be a connected, simple, and finite graph. Let f be a bijective function of labeling on graph G from the edge set $E(G)$ to natural number up to the number of edges of G . A rainbow vertex antimagic labeling of graph G is a function f under the condition all internal vertices of a path $u - v, \forall u, v \in V(G)$ have different weight (denoted by $w(u)$), where $w(u) = \sum_{uu' \in E(G)} f(uu')$. If G has a rainbow vertex antimagic labeling, then G is a rainbow vertex antimagic coloring, where the every vertex is assigned with the color $w(u)$. The $rvac(G)$ is a notation of rainbow vertex antimagic connection number of graph G which means the minimum colors taken over all rainbow vertex antimagic coloring induced by rainbow vertex antimagic labeling of graph G . The results of this research are the exact value of the rainbow vertex antimagic connection number of star (S_n), double star (DS_n), and broom graph ($Br_{n,m}$).

1. Introduction

Graph coloring is the famous topics in graph theory. Many types of colorings and connectivity measures have been study with many researchers. The rainbow antimagic coloring is a combination of three concept namely coloring, labeling and connectivity, and this topics was first defined by Septory et al [14]. Many famous researchers have worked on antimagic rainbow coloring [1, 14]. They found new results and it has many contributions in graph theory, especially in rainbow coloring or rainbow connection.

All graphs considered in this paper are simple, finite and undirected [8]. We follow the terminology and notation about the graph by Chartrand et al [2]. The concept of rainbow connection [17] introduced by Chartrand et al [3, 4]. A path is rainbow if no two edges are the same color. If every two vertices of an edge-coloring graph G are connected by a rainbow path, the graph is rainbow connected. A rainbow connection coloring is an edge-coloring in which G is rainbow connected. Rainbow vertex-connection is one of several interesting variants of the rainbow connection concept [5, 6, 9].

Let G be a nontrivial connected graph with an edge-coloring $c : E(G) \rightarrow \{1, 2, 3, \dots, k\}$, $k \in \mathbb{N}$, where adjacent edges can be colored the same. A rainbow path P of G is one in which no two edges of P are the same color. The graph G is said to be rainbow-connected if there is a rainbow $u - v$ path between any two vertices u and v of G [7, 12].

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Similar to the concept of the rainbow connection number. A vertex-colored graph is rainbow vertex-connected if any two vertices are connected by a path whose internal vertices have distinct colors. If k colors are used, G becomes rainbow k -vertex-connected. The rainbow vertex-connection number of a connected graph G , represented by $rvc(G)$, is the smallest number of colors required to make G rainbow vertex-connected. According to Krivelevich and Yuster [11], the lower bound for $rvc(G)$ is $rvc(G) \geq diam(G) - 1$, where $diam(G)$ is the diameter of graph G . There are various results for rainbow vertex-connection numbers and rainbow connection numbers. The results of rainbow vertex connection number can be seen in [10, 15].

Marsidi et al. [13] develop a new concept in rainbow coloring namely rainbow vertex antimagic coloring. Let $G(V(G), E(G))$ be a connected graph of size q and $f : E(G) \rightarrow \{1, 2, 3, \dots, q\}$ be a labeling of a graph G . The function f is called a rainbow vertex antimagic labeling if for any two vertices u and v in $V(G)$ all internal vertices in path $u - v$ have different weight. The vertex weight denoted by $w_f(u)$ for every $u \in V(G)$, where $w_f(u) = \sum_{uu' \in E(G)} f(uu')$. If each edge of G is assigned with the color of the vertex weight $w_f(u)$, then G admits a rainbow vertex antimagic coloring. The $rvac(G)$ is a notation of rainbow vertex antimagic connection number of graph G which means the minimum colors taken over all rainbow vertex antimagic coloring induced by rainbow vertex antimagic labeling of graph G . The results of this research are the exact value of the rainbow vertex antimagic connection number of star (S_n), double star (DS_n), and broom graph ($Br_{n,m}$).

2. Preliminaries on rainbow vertex antimagic coloring

In previous studies, Marsidi et al.[2] have determined the lower bound of rainbow vertex antimagic connection number as follows. The resulted lower bound can be used to determine the rainbow vertex antimagic connection number for any graph.

Remark 1 [13] Let G be a connected graph, $rvac(G) \geq rvc(G)$.

Based on Remark 2, we determined specifically the lower bound of rainbow vertex antimagic connection number for graphs belonging to the family tree graph. The lower bound of the rainbow vertex antimagic connection number is presented in Lemma 1.

Lemma 1 Let T be a tree graph of order p has l pendant vertices,

$$rvac(T) \geq \begin{cases} rvc(T), & \text{if } l < p - l \\ l + 1, & \text{if } l \geq p - l. \end{cases}$$

Proof. Let T be a tree graph of order p has l pendant vertices and f be function under the antimagic rainbow edge labeling. Since $l < p - l$, then base on Remark 2 it gives $rvac(T) \geq rvc(T)$. Let $v \in V(T)$ is a pendant vertex. Since vertex v is assigned with the color $w(v)$, then $w(v) = f(e)$ where e is incident edge with v . Hence all the pendant vertices receive distinct colors. Furthermore, for $l \geq p - l$ any non-pendant vertex v' incident with an edge e' with $f(e') = s$, the color assigned to v' is larger than s . Hence the number of colors in the vertex coloring induced by f is at least $l + 1$. It concludes that $rvac(T) \geq l + 1$. \square

3. Results and Discussions

The results of this study are new theorems regarding the rainbow vertex antimagic connection number. The graphs studied in this paper are family of tree, namely star, double star, and broom.

Theorem 1 Let S_n be a star graph. For every positive integer $n \geq 3$, $rvac(S_n) = n + 1$.

Proof. The star graph S_n is a connected graph with vertex set $V(S_n) = \{\alpha, u_i : 1 \leq i \leq n\}$ and edge set $E(S_n) = \{\alpha u_i : 1 \leq i \leq n\}$. Star graph has n pendant vertices and one vertex (center) apart from pendant vertices. Since $l > p - l$ or $n > 1$, based on Lemma 1 we have $rvac(S_n) \geq l + 1 = n + 1$ as lower bound. Further, we will determine the upper bound of $rvac(S_n)$ by deriving the bijection of edge labels as follows.

$$g_1(\alpha u_i) = i : 1 \leq i \leq n$$

The vertex weight can be determined from the edge label function, such that we obtain:

$$w(u_i) = i : 1 \leq i \leq n$$

$$w(\alpha) = \frac{n(n+1)}{2}$$

Based on the vertex weights above, we know that the graph S_n has $n + 1$ different weights. Since every vertex $u \in V(S_n)$ is assigned with the color $w(u)$, then internal vertices for every two different vertices have different weights. For more detail, it can be seen in Table 1. Suppose that $u, u' \in V(S_n)$, there are two cases of rainbow paths in S_n . There are as follows.

Table 1. The Rainbow Vertex of $u - u'$ Path of S_n

Case	u	u'	Rainbow Vertex Coloring $u - u'$
1	α	u_i	αu_i
2	u_i	u_i	u_i, α, u_i

From Table 1, we can conclude that the graph S_n is rainbow vertex antimagic coloring. Thus, we obtain $rvac(S_n) = n + 1$. \square

Theorem 2 Let $DS_{n,m}$ be a double star graph. For every positive integer $n, m \geq 2$,

$$rvac(DS_{n,m}) = n + m + 2.$$

Proof. The double star graph $DS_{n,m}$ is a connected graph with vertex set $V(DS_{n,m}) = \{\alpha, \beta, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(DS_{n,m}) = \{\alpha\beta, \alpha u_i, \beta v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$. Double star graph has $n + m$ pendant vertices and 2 vertex apart from pendant vertices. Since $l > p - l$ or $n + m > 2$, based on Lemma 1 we have $rvac(DS_{n,m}) \geq l + 1 = n + m + 1$. Since vertex α and β are always be internal vertices for every two vertices u_i and v_j , then vertex α and β must be receive the different weight. It gives $rvac(DS_{n,m}) \geq n + m + 2$. Further, we will determine the upper bound of $rvac(DS_{n,m})$ by deriving the bijection of edge labels as follows.

$$g_2(\alpha u_i) = i : 1 \leq i \leq n$$

$$g_2(\beta v_j) = n + j : 1 \leq j \leq m$$

$$g_2(\alpha\beta) = n + m + 1$$

The vertex weight can be determined from the edge label function, such that we obtain:

$$w(u_i) = i : 1 \leq i \leq n$$

$$w(v_j) = n + j : 1 \leq j \leq m$$

$$w(\alpha) = \frac{n(n+1)}{2} + n + m + 1$$

$$w(\beta) = nm + \frac{m(m+1)}{2} + n + m + 1$$

Based on the vertex weights above, we know that the graph $DS_{n,m}$ has $n + m + 2$ different weights. Since every vertex $u \in V(DS_{n,m})$ is assigned with the color $w(u)$, then internal vertices for every two different vertices have different weights. For more detail, it can be seen Table 2. Suppose that $u, u' \in V(DS_{n,m})$, there are five cases of rainbow paths in $DS_{n,m}$. There are as follows.

Table 2. The Rainbow Vertex of $u - u'$ Path of $DS_{n,m}$.

Case	u	u'	Rainbow Vertex Coloring $u - u'$
1	α	u_i	αu_i
2	u_i	u_i	u_i, α, u_i
3	β	v_j	β, v_j
4	v_j	v_j	v_j, β, v_j
5	u_i	v_j	u_i, α, β, v_j

From the Table 2, we can conclude that the graph $DS_{n,m}$ is rainbow vertex antimagic coloring. Thus, we obtain $rvac(DS_{n,m}) = n + m + 2$.

Theorem 3 Let $Br_{4,m}$ be a broom graph. For every positive integer $m \geq 2$,

$$rvac(Br_{4,m}) = m + 2.$$

Proof. The broom graph $Br_{4,m}$ is a connected graph with vertex set $V(Br_{4,m}) = \{u_i, v_j : 1 \leq i \leq 4, 1 \leq j \leq m\}$ and edge set $E(Br_{4,m}) = \{u_1u_2, u_2u_3, u_3u_4, u_4v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$. Graph $Br_{4,m}$ has $m + 1$ pendant vertices. Since $l \geq p - l$ or $m + 1 \geq 3$, based on Lemma 1 we have $rvac(Br_{4,m}) \geq l + 1 = m + 1 + 1 = m + 2$. Further, we will determine the upper bound of $rvac(Br_{4,m})$ by deriving the bijection of edge labels as follows.

$$g_3(u_1u_2) = 3$$

$$g_3(u_2u_3) = 1$$

$$g_3(u_3u_4) = 2$$

$$g_3(u_4v_j) = 3 + j : 1 \leq j \leq m$$

The vertex weight can be determined from the edge label function, such that we obtain:

$$w(u_1) = 3$$

$$w(u_2) = 4$$

$$w(u_3) = 3$$

$$w(u_4) = 2 + 3m + \frac{m(m+1)}{2}$$

$$w(v_j) = 3 + j : 1 \leq j \leq m$$

Based on the vertex weights above, we know that the graph $Br_{4,m}$ has $m+2$ different weights. Since every vertex $u \in V(Br_{4,m})$ is assigned with the color $w(u)$, then internal vertices for every two different vertices have different weights. For more detail, it can be seen Table 3. Suppose that $u, u' \in V(Br_{4,m})$, there are four cases of rainbow paths in $Br_{4,m}$. There are as follows.

Table 3. The Rainbow Vertex of $u - u'$ Path of $Br_{4,m}$.

Case	u	u'	Condition	Rainbow Vertex Coloring $u - u'$
1	u_i	u_j	$i < j$	$u_i, u_{i+1}, u_{i+2}, \dots, u_j$
2	u_i	u_j	$i > j$	$u_i, u_{i-1}, u_{i-2}, \dots, u_j$
3	u_i	v_j	$1 \geq i \geq 4$, and $1 \geq j \geq m$	u_i, \dots, u_1, v_j
4	v_i	v_j		v_i, u_1, v_j

From the Table 3, we can concludes that graph $Br_{4,m}$ is rainbow vertex antimagic coloring. Thus, we obtain $rvac(Br_{4,m}) = m + 2$. \square

Theorem 4 Let $Br_{n,3}$ be a broom graph. For every positive integer $n \geq 3$,

$$rvac(Br_{n,3}) \geq \begin{cases} 5, & \text{if } n = 3, 4, 5 \\ n - 1, & \text{if } n \geq 6. \end{cases}$$

Proof. The broom graph $Br_{n,3}$ is a connected graph with vertex set $V(Br_{n,3}) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq 3\}$ and edge set $E(Br_{n,3}) = \{u_i u_{i+1}, u_1 v_j : 1 \leq i \leq n-1, 1 \leq j \leq 3\}$. Graph $Br_{n,3}$ has $3+1 = 4$ pendant vertices and $diam(Br_{n,3}) = n$. For $n = \{3, 4, 5\}$, since $l \geq p-l$ then based on Lemma 1 we have $rvac(Br_{n,3}) \geq l+1 = 4+1 = 5$. For $n \geq 6$, since it has $p-l > l$ or $n-1 > 4$, then based on Lemma 1 we have $rvac(Br_{n,3}) \geq rvc(Br_{n,3}) = diam(Br_{n,3}) - 1 = n - 1$. Further, we will determine the upper bound of $rvac(Br_{n,3})$ by deriving the bijection of edge labels as follows. For $n = \{3, 4, 5, 6\}$, we show the edge labels in Figure 1.

Based on Figure 1, it is easy to see that graph $Br_{n,3} : n = \{3, 4, 5, 6\}$ have 5 different weights and internal vertices for every two different vertices have different weights. For $n \geq 6$, there are two cases, namely for n odd and n even as follows.

Case 1. For n Odd

$$g_4(u_i u_{i+1}) = \begin{cases} \frac{i+1}{2}, & \text{if } i \text{ odd} \\ \frac{n-1+i}{2}, & \text{if } i \text{ even} \end{cases}$$

$$g_4(u_1 v_j) = n - 1 + j : 1 \leq j \leq 3$$

The vertex weight can be determined from the edge label function, such that we obtain:

$$w(u_1) = 3n + 4$$

$$w(u_i) = \frac{n-1}{2} + i : 2 \leq i \leq n-1$$

$$w(u_n) = n - 1$$

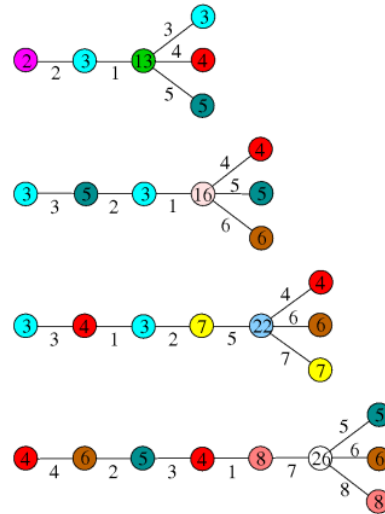


Figure 1. The Edge Labels on $Br_{n,3} : n = \{3, 4, 5, 6\}$.

$$w(v_j) = n - 1 + j : 1 \leq j \leq 3$$

Case 2. For n Even

$$g_4(u_i u_{i+1}) = \begin{cases} \frac{i}{2}, & \text{if } i \text{ even} \\ \frac{n-3+i}{2}, & \text{if } i \text{ odd} . \end{cases}$$

$$g_4(u_1 v_j) = n - 2 + j : 1 \leq j \leq 3$$

$$g_4(u_1 u_2) = n + 2$$

The vertex weight can be determined from the edge label function, such that we obtain:

$$w(u_1) = 4n + 2$$

$$w(u_2) = n + 3$$

$$w(u_i) = \frac{n-4}{2} + i : 3 \leq i \leq n-1$$

$$w(u_n) = n - 2$$

$$w(v_j) = n - 2 + j : 1 \leq j \leq 3$$

Based on the vertex weights above, we know that the graph $Br_{n,3}$ has 5 different weights if $n = \{3, 4, 5\}$ and $n - 1$ if $n \geq 6$. Since every vertex $u \in V(Br_{n,3})$ is assigned with the color $w(u)$, then internal vertices for every two different vertices have different weights. For more detail, it can be seen Table 4. Suppose that $u, u' \in V(Br_{n,3})$, there are four cases of rainbow paths in $Br_{n,3}$. There are as follows.

From the Table 4, we can conclude that the graph $Br_{n,3}$ is rainbow vertex antimagic coloring. Thus, we obtain $rvac(Br_{n,3}) = 5$ if $n = \{3, 4, 5\}$ and $rvac(Br_{n,3}) = n - 1$ if $n \geq 6$. \square

Table 4. The Rainbow Vertex of $u - u'$ Path of $Br_{n,3}$.

Case	u	u'	Conditions	Rainbow Vertex Coloring $u - u'$
1	u_i	u_j	$i < j$	$u_i, u_{i+1}, u_{i+2}, \dots, u_j$
2	u_i	u_j	$i > j$	$u_i, u_{i-1}, u_{i-2}, \dots, u_j$
3	u_i	v_j	$1 \geq i \geq n$, and $1 \geq j \geq 3$	u_i, \dots, u_1, v_j
4	v_i	v_j		v_i, u_1, v_j

4. Concluding Remarks

We have obtained the rainbow vertex antimagic connection number of family tree graphs, namely star, double star, and broom graphs. Finding the an exact values of rainbow vertex antimagic connection number of any graphs is considered to be NP-problem. Therefore, we have the following open problems.

Open Problem 1 Determine the rainbow vertex antimagic connection number of any other family tree graphs.

Open Problem 2 Determine the rainbow vertex antimagic connection number of any family graphs, such as regular graphs, almost regular graphs, and others.

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